MATH 118: Practice Midterm 2 Key

Name: ______________________________________

Directions:

* Show your thought process (commonly said as ”show your work”) when solving each problem for full credit.
* If you do not know how to solve a problem, try your best and/or explain in English what you would do.
* Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
		80

1. Short answer questions:

   (a) Given the function
   $$F(x) = \sqrt{x^2+1}$$
   find two functions f, g where $f \circ g = F$. You are not allowed to choose $f(x) = x$ or $g(x) = x$.

   f is the outside and g is the inside.

   $\boxed{\text{Choose } f(x) = \sqrt{x} \text{ and } g(x) = x^2+1.}$

   (b) True or False: Whenever you see a negative square root, such as $\sqrt{-5}$, you should immediately pull out the $-$ and write $i\sqrt{5}$.

   $\boxed{\text{True.}}$

   (c) True or False: if $x = 3$ is a zero of the polynomial $P(x)$, then $(x+3)$ is a factor of $P(x)$.

   $\boxed{\text{False.}}$

   According to the definition of zeros, if $x = 3$ is a zero, then $(x-3)$ is a factor of $P(x)$, not $(x+3)$.

   (d) True or False: If $f(x) = x^2$, then $f(x+h) = x^2 + h$. If not, what should it be instead?

   $\boxed{\text{False. It should be } f(x+h) = (x+h)^2.}$

2. Solve the following equations and inequalities:

(a) $\sqrt{2x+1}+1=x$

Isolate root then square. Expand and treat like a quadratic equation $ax^2+bx+c=0$

$$\begin{aligned}\sqrt{2x+1}+1&=x\\ \sqrt{2x+1}&=x-1\\ \left(\sqrt{2x+1}\right)^2&=(x-1)^2\\ 2x+1&=x^2-2x+1\\ 0&=x^2-4x\end{aligned}$$

Use Method 1, factoring.

$$\begin{aligned}x^2-4x&=0\\ x(x-4)&=0\\ x=0 &\qquad x-4=0\\ x=0 &\qquad x=4\end{aligned}$$

Now check solutions because it is a root equation.

For $x=0$:

- LHS: $\sqrt{2\cdot 0+1}+1=1+1=2$
- RHS: 0

$x=0$ is not a solution.

For $x=4$:

- LHS: $\sqrt{2\cdot 4+1}+1=3+1=4$
- RHS: 0

$\boxed{\text{x = 4 is the only solution.}}$

(b) $x^2+3x+2=x$

Put into quadratic equation form and use quadratic formula.

$$\begin{aligned}x^2+3x+2&=x\\ x^2+2x+2&=0\end{aligned}$$

With $a = 1, b = 2, c = 2$ we have

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} \\ &= \frac{-2 \pm \sqrt{4 - 8}}{2} \\ &= \frac{-2 \pm \sqrt{-4}}{2} \\ &= \frac{-2 \pm i\sqrt{4}}{2} \\ &= \frac{-2 \pm 2i}{2} \\ &= \frac{2 \cdot (-1 \pm i)}{2} \\ &= \boxed{-1 \pm i} \end{aligned}$$

(c) $1 \leq -2x + 7 < 9$

Subtract 7, divide by -2. Don't forget to flip the inequalities.

$$\begin{aligned} 1 \leq -2x + 7 < 9 \\ -6 \leq -2x < 2 \\ 3 \geq x > -1 \end{aligned}$$

or reading right-to-left, $\boxed{-1 < x \leq 3}$

(d) $\dfrac{\frac{4}{x^2} - 1}{x} = 0$

Deal with denominators. Similar to problem in lecture.

$$\frac{\frac{4}{x^2} - 1}{x} = 0 \qquad \text{Initial expression}$$

$$x \cdot \frac{\frac{4}{x^2} - 1}{x} = 0 \cdot x \qquad \text{Multiply by LCD, } x$$

$$\cancel{x} \cdot \frac{\frac{4}{x^2} - 1}{\cancel{x}} = 0 \qquad \text{Frac. Prop. 5}$$

$$\frac{4}{x^2} - 1 = 0 \qquad \text{Rewrite for better optics on our problem}$$

$$x^2 \left(\frac{4}{x^2} - 1 \right) = 0 \cdot x^2 \qquad \text{Multiply by LCD, } x^2$$

$$x^2 \cdot \frac{4}{x^2} - x^2 \cdot 1 = 0 \qquad \text{Dist. Law}$$

$$\cancel{x^2} \cdot \frac{4}{\cancel{x^2}} - x^2 = 0 \qquad \text{Frac. Prop. 5}$$

$$4 - x^2 = 0 \qquad \text{Rewrite for better optics on our problem}$$

$$4 = x^2 \qquad \text{Get in form } x^2 = c$$

$$\pm\sqrt{4} = \sqrt{x^2} \qquad \text{Take square root}$$

$$\boxed{x = \pm 2} \qquad \text{Done}$$

3. Perform the given instruction.

(a) Suppose $g(x) = -|-2x+4|+3$. Write the order of transformations you would use to transform $f(x) = |x|$ into $g(x)$.

We have

$$\begin{aligned} g(x) &= -|-2x+4|+3 \\ &= -|-2(x-2)|+3 \end{aligned}$$

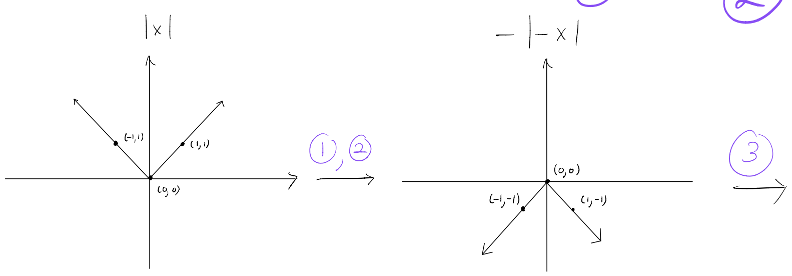
x-axis reflection ①

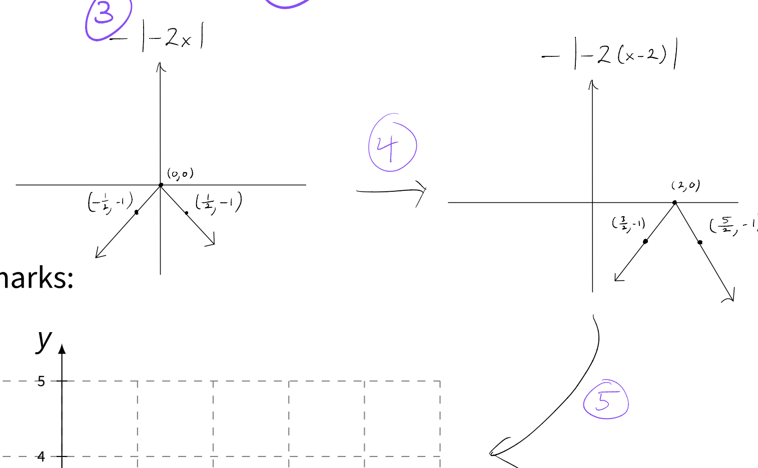
y-axis reflection ②

h shrink by a factor of $\frac{1}{2}$ ③

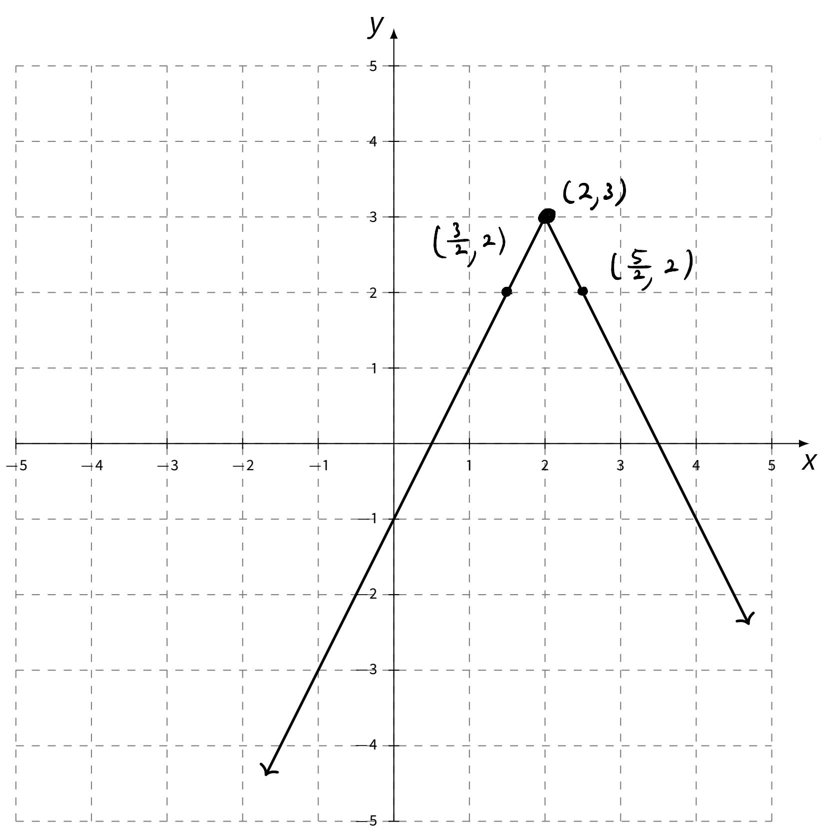
h shift right 2 ④

v shift up 3 ⑤





(b) Now graph it and choose your tick marks:



4. Find the domain for each of the following functions:

(a) $f(x) = x^8 - 2x^7 + 4x^2 - 2$

$\boxed{\mathbb{R}}$ because there are no problems to remove.

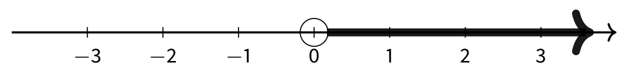
(b) $h(x) = \sqrt{x} + \dfrac{1}{x}$

1 Problems.

i. Solve $x = 0$. Done.

ii. Solve $x < 0$. Done.

2 Remove problems.



Domain: $\boxed{(0, \infty)}$

(c) $f(x) = \dfrac{1}{x^2 - 3x + 2}$

1 Problems.

i. Solve $x^2 - 3x + 2 = 0$. We have

$1 \quad x \quad -2$
$1 \quad\quad -1$

$$(x-2)(x-1) = 0$$

$$x - 2 = 0 \qquad x - 1 = 0$$

$$x = 2 \qquad x = 1$$

ii. N/A, no root.

2 Remove problems.

Domain: $\boxed{(-\infty, 1) \cup (1, 2) \cup (2, \infty)}$

5. Consider $f(x) = 1 - x$ and $g(x) = x^2 - x$. Expand **and simplify** the following:

(a) $f(x) - 3g(x)$

We have

$$\begin{aligned} f(x) - 3g(x) &= 1 - x - 3(x^2 - x) \\ &= 1 - x - 3x^2 + 3x \\ &= -3x^2 + 2x + 1 \\ &= -(3x^2 - 2x - 1) && \text{GCF, common factor of } -1 \\ &= \boxed{-(3x+1)(x-1)} \end{aligned}$$

3 1
1 −1

(b) $f(x)g(x)$

We have

$$\begin{aligned} f(x)g(x) &= (1-x)(x^2-x) \\ &= x^2(1-x) - x(1-x) && \text{Dist. Law} \\ &= x^2 - x^3 - x + x^2 && \text{Dist. Law} \\ &= -x^3 + 2x^2 - x \\ &= -x(x^2 - 2x + 1) && \text{GCF, common factor of } -x \\ &= \boxed{-x(x-1)^2} && A^2 - 2AB + B^2 = (A-B)^2 \end{aligned}$$

(c) $f \circ g$

We have

$$\begin{aligned} (f \circ g)(x) &= f\big(g(x)\big) \\ &= f(x^2 - x) \\ &= 1 - (x^2 - x) \\ &= \boxed{-x^2 + x + 1} \end{aligned}$$

(d) $g(x+h) - g(x)$

We have

$$\begin{aligned} g(x+h) - g(x) &= (x+h)^2 - (x+h) - (x^2 - x) \\ &= x^2 + 2xh + h^2 - x - h - x^2 + x && \text{Expanding everything} \\ &= 2xh + h^2 - h \\ &= \boxed{h(2x + h - 1)} \end{aligned}$$

6. Perform the given instruction.

   (a) Draw a graph which satisfies the following:

      i. Increasing on $(-\infty, -2) \cup (-1, 1) \cup (2, \infty)$

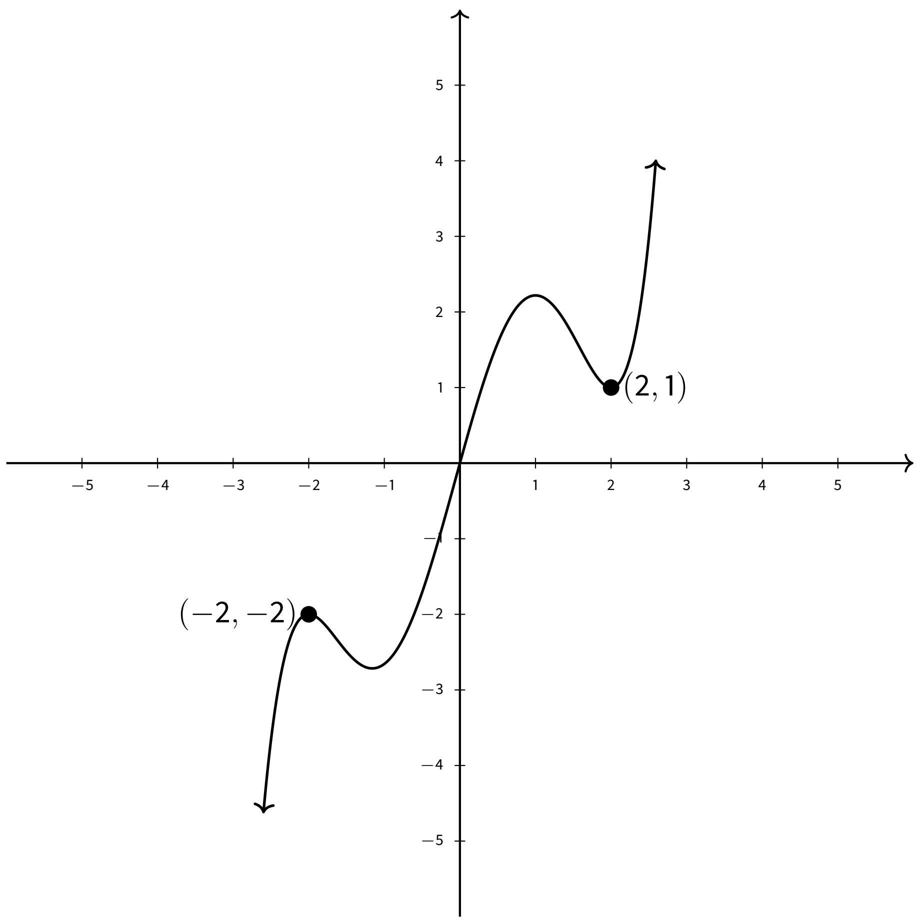
      ii. Decreasing on $(-2, -1) \cup (1, 2)$

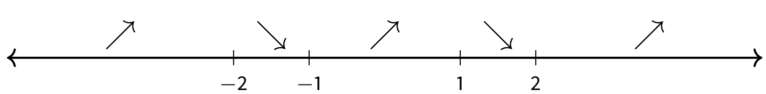
      iii. Local maxima of $f(-2) = -2$

      iv. Local minima of $f(2) = 1$

      v. $f(0) = 0$

   Answers may vary.





(b) Put $f(x) = x^2 - 8x + 8$ into standard form. What is the vertex?

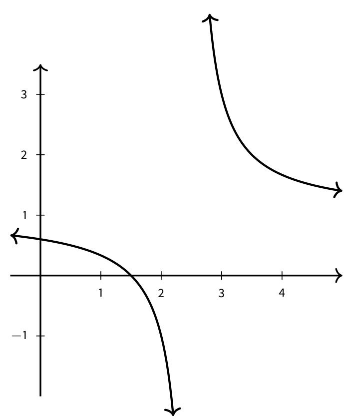
$b = -8$. So $\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = (-4)^2 = 16$. Adding and subtracting:

$$\begin{aligned} f(x) &= x^2 - 8x + 8 \\ &= x^2 - 8x + 16 - 16 + 8 && \text{Add and subtract} \\ &= (x^2 - 8x + 16) - 16 + 8 && \text{Group first three terms} \\ &= (x-4)^2 - 16 + 8 && A^2 - 2AB + B^2 = (A-B)^2 \\ &= \boxed{(x-4)^2 - 8} \end{aligned}$$

$\boxed{\text{The vertex is } (4, -8).}$

(c) Find the inverse of the function $f(x) = \dfrac{2x-3}{2x-5}$. You may use the fact that $f(x)$ is one-to-one.

1 It is one to one:



2 Let $y = \dfrac{2x-3}{2x-5}$

3 Solve for x.

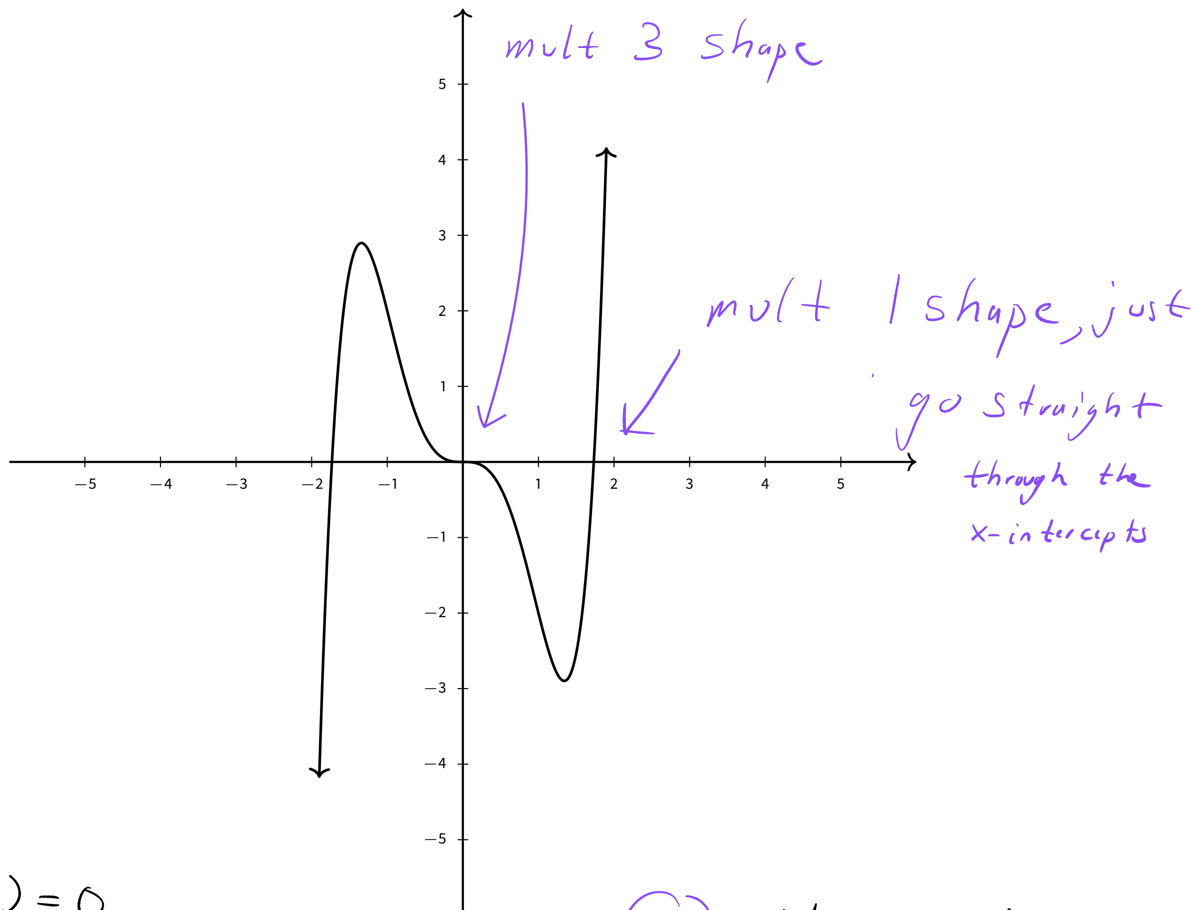
$$\begin{aligned} y &= \frac{2x-3}{2x-5} \\ (2x-5)\cdot y &= \frac{2x-3}{\cancel{(2x-5)}} \cdot \cancel{(2x-5)} \\ 2xy - 5y &= 2x - 3 \\ 2xy - 2x &= 5y - 3 \\ x(2y-2) &= 5y - 3 \\ x &= \frac{5y-3}{2y-2} \end{aligned}$$

4 Swap x and y: $y = \dfrac{5x-3}{2x-2}$

$\boxed{\text{Result } : f^{-1}(x) = \dfrac{5x-3}{2x-2}}$

7. Suppose $P(x) = x^5 - 3x^3$. Sketch a graph of $P(x)$ using the four step process.

4



1 Solve $P(x) = 0$

$$x^5 - 3x^3 = 0$$

$$x^3(x^2 - 3) = 0$$

$x^3 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{0}$

$x = 0$

mult 3

$x^2 = 3$

$\sqrt{x^2} = \pm\sqrt{3}$

$x = \pm\sqrt{3}$

mult 1 because

$x = \sqrt{3} \rightarrow (x - \sqrt{3})$ is a factor

$x = -\sqrt{3} \rightarrow (x + \sqrt{3})$ is a factor

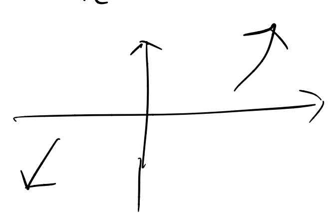
$\rightarrow x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

2 No need, use multiplicity.

3 leading term

x^5 odd degree

$a_n = 1 > 0$



8. Divide $P(x) = 4x^3 + 7x + 9$ by $D(x) = 2x + 1$. Write your answer in the form of the Division Algorithm.

   Not on midterm!